INTRODUCTION
There is increasing concern about the likelihood of airport vicinity pollution levels violating ambient air quality regulations as airport usage continues to grow. Recent research has shown that, in the EU regulatory context, the annual average NO$_2$ limit value is one of the primary constraints on traffic growth. Well-developed modeling tools such as EDMS and ADMS-Airport are able to generate hour-by-hour air quality predictions of considerable accuracy. These approaches are appropriate for detailed investigations, but it may be beneficial to develop an operational screening model for expansion plans and policy options. Such a model will be used in the UK Aviation Integrated Modelling Project, based in Cambridge, which aims to simulate worldwide aviation, environmental and economic interactions to a 2050 timeframe (Reynolds et al., 2007).

REGULATORY CONTEXT
There is considerable variation in ambient air quality standards across the world (Watkins et al., 2004). For example, the EU and US annual average limit values for NO$_2$ are 40 and 100 µg/m$^3$ respectively. Recent studies on airport air quality include those by Unal et al. (2005) and Schürmann et al. (2007). The Project for the Sustainable Development of Heathrow (PSDH) is perhaps the most comprehensive recent study (UK Department for Transport, 2006). Its purpose was to evaluate the quality of models that could be used to analyze possible UK airport developments subject to the ambient air quality regulatory constraints of EU Council Directive 1999/30/EC.

Of the various regulated pollutants, in the context of airports NO$_2$ and particulate matter (PM) are of particularly significant concern. PSDH found that the most immediate air quality constraint is the NO$_2$ annual average limit value. PM contributions due to aircraft were overwhelmed by road traffic sources, some of which may be regarded as airport-accountable.

OPERATIONAL MODELLING APPROACHES
There are a number of widely used operational atmospheric dispersion models applied to airports, for example:
- ADMS-Airport, based on ADMS for dispersion calculations (Carruthers et al., 1994);
- EDMS (CSSI, Inc., 2004), based on AERMOD (Cimorelli et al., 2004); and
- LASPORT, based on LASAT (Janicke Consulting).

ADMS and AERMOD can be described as Gaussian plume models, while LASAT is a Lagrangian model. All have been widely applied and evaluated against experimental results, but require long run times and have significant meteorological, morphometrical and emissions data input requirements. We therefore aim to develop a simple method for predicting airport local air quality, which is appropriate as a screening model. We only consider long-term averaged pollutant concentrations, as they are of most immediate regulatory interest (for NO$_2$), and aim for the minimum possible data input requirements and rapid execution.
ASSUMPTIONS AND RESTRICTIONS
The assumptions and restrictions that will be drawn upon for the purposes of this extended abstract are outlined in Table 1.

Table 1. Outline of assumptions and restrictions in proposed screening model.

<table>
<thead>
<tr>
<th>Assumption/Restriction</th>
<th>Rationale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emissions represented as ground level sources, neglect influence of emissions at altitude</td>
<td>As a baseline we neglect all above-ground emissions to reduce user burden in the context of a simple screening model, Wayson and Fleming (2000) results show impact of emissions at altitude</td>
</tr>
<tr>
<td>Buoyancy, trailing vortices, downwash, jet momentum neglected</td>
<td>Interpretation of CERC study for PSDH indicates this will be acceptable for a screening model, reduced user burden</td>
</tr>
<tr>
<td>Dispersion considered for conserved scalars only</td>
<td>Appropriate for PM over time scales of interest, NOX chemistry can be applied empirically (e.g. Jenkin, 2004)</td>
</tr>
<tr>
<td>Single roughness length</td>
<td>Reduced user burden, average concentrations would have ~15% error if z0 had a factor of two error, Hanna and Britter (2002)</td>
</tr>
<tr>
<td>Very low wind conditions neglected</td>
<td>Use of minimum wind speed</td>
</tr>
<tr>
<td>Neutral conditions assumed</td>
<td>Assumed for simplicity and reduced user burden, acceptable given typically</td>
</tr>
<tr>
<td>Only long-term averages can be calculated</td>
<td>Corresponds to most immediate regulatory constraint</td>
</tr>
<tr>
<td>Flat urban airport</td>
<td>Often applicable, required for other assumptions</td>
</tr>
</tbody>
</table>

MODEL FORMULATION
A location's annual or seasonal wind is often represented in a statistical format graphically as a wind rose. We will interpret wind rose data as a joint probability density function \( p(\theta, u_r) \), where \( \theta \) is the wind direction, and \( u_r \) is a reference wind speed at a constant reference height, \( z_r = 10 \) m for most airports. The annual average concentration is given by

\[
\langle \chi(x, y) \rangle = \int \int \sum_{i=1}^{N} p(\theta, u_r) \chi(x, y, \theta, u_r) \, du_r \, d\theta
\]

where there are \( N \) sources and \( \chi(\ldots) \) is an appropriate dispersion kernel. The advantage of this approach is that it directly yields the mean value required. This is in the contrast to hour-by-hour meteorological and emissions data inputs and dispersion calculations, followed by post-processing to determine the average concentration field performed by most models.

Given the assumptions outlined, we only need friction velocity, \( u_* \), and surface roughness, \( z_0 \), to characterize dispersion approximately. Since \( z_r \) is within the log-law region of the boundary layer to a reasonable approximation, we can define a dimensionless velocity as

\[
\hat{u} = \frac{u_r}{u_*} = \frac{\ln(z_r / z_0)}{\kappa}
\]

where \( \kappa = 0.4 \). In the current framework of assumptions and restrictions, \( \hat{u} \) is constant for a given location, so \( u_* \) is simply calculated from Eq. (2). Substitution of typical urban numbers with Dyer-Hicks stability terms in the denominator of Eq. (2) demonstrates the low influence of stability given the assumptions and restrictions outlined. Typically \( \hat{u} = 10 \).

For the purposes of this extended abstract, we will describe the simplest possible application of this approach: a point source with a uniform boundary layer. Assuming that lateral and vertical dispersion are decoupled, as for example described by Calder (1952) or Pasquill and Smith (1983), we can then represent concentrations by a lateral and vertical component as in

\[
\chi(x, y, z)/Q = \chi_l(x, y)\chi_v(x, z),
\]

where \( Q \) is the pollutant emission rate. The appropriate two-dimensional formula found by Calder (1952) reads
\[
\chi_v(x, z) = \frac{1}{k u_x} \exp \left( - \frac{\bar{u} z}{k u_x} \right),
\]

where \( \bar{u} \) is the (uniform) wind speed. We are only interested in the ground-level concentrations \( (z = 0) \). For lateral dispersion we chose the well-documented Gaussian profile

\[
\chi_l(x, y) = \frac{1}{\sigma_x \sqrt{2\pi}} \exp \left( - \frac{y^2}{2\sigma_x^2} \right),
\]

where for neutral conditions and urban roughness \( \sigma_y = bx \) \( (b = 0.16) \) is appropriate over short ranges (e.g., Hanna and Britter, 2002).

Direct application of Eq. (5) in Eq. (1) requires appropriate coordinate rotations. In practice this would require a double numerical integration, over wind direction and speed. However, a computational shortcut can be made by considering a normalized concentration

\[
\hat{\chi} = \chi u_r / Q,
\]

which allows us to write

\[
\langle \chi(x, y) \rangle = \sum_{i=1}^{N} Q_i \int \hat{\chi}(x, y; \theta) \int u_r^{-1} p(\theta, u_r) du_r d\theta.
\]

We define an average inverse wind speed per wind direction as

\[
\langle u_r^{-1}(\theta) \rangle = \int u_r^{-1} p(u_r, \theta) du_r
\]

and the marginal probability of any particular wind direction as

\[
p(\theta) = \int p(\theta, u_r) du_r,
\]

so to find

\[
\langle \chi(x, y) \rangle = \sum_{i=1}^{N} Q_i \int \hat{\chi}(x, y; \theta) \langle u_r^{-1}(\theta) \rangle p(\theta) d\theta.
\]

This means that rather than two nested numerical integrations, we have two sequential numerical integrations. For example a nested grid of 100 by 100 with 20 wind speed bins and 36 directions would normally require \( 100 \times 100 \times 20 \times 36 = 7.2 \) million dispersion kernel evaluations by direct evaluation of Eq. (1). By comparison Eq. (8) requires 36 integrations over 20 points, then \( 100 \times 100 \times 36 = 360,000 \) dispersion kernel evaluations.

Equations (1) and (8) should yield identical results. A further saving can be made by introducing an approximation relating to the importance of lateral dispersion over long-term averages. Consider application of Eq. (8) as a ‘beam’ of pollution sweeping over a particular receptor, which accumulates contributions according to its ‘strength’ \( \langle u_r^{-1}(\theta) \rangle p(\theta) \). If this changes slowly over the width of the ‘beam’, then the specific lateral dispersion profile is not important. To see this, we note that Eq. (8) can be written

\[
\langle \chi(x, y) \rangle = \sum_{i=1}^{N} Q_i \int \chi_i(x, y; \theta) \hat{\chi}_r(x; \theta) \langle u_r^{-1}(\theta) \rangle p(\theta) d\theta \quad (\hat{\chi}_r = \chi_i u_r). \tag{9}
\]

Over the width of the ‘beam’ we now assume \( \langle u_r^{-1}(\theta) \rangle p(\theta) \) is almost constant and write \( dy = xd\theta \), which is a good approximation where the concentrations are highest. It follows

\[
\langle \chi(x, y) \rangle = \sum_{i=1}^{N} Q_i p(\theta) \langle u_r^{-1}(\theta) \rangle \hat{\chi}_r(x; \theta) \int \chi_i(x, y; \theta) dy. \tag{10}
\]

Noting that by continuity the remaining integral, for any \( \chi_i \), must be unity, we have finally

\[
\langle \chi(R, \theta) \rangle = \sum_{i=1}^{N} Q_i p(\theta) \langle u_r^{-1}(\theta) \rangle \frac{\hat{u}}{k R_i^2}, \tag{11}
\]

where polar coordinates \( (R, \theta) \) are measured from each point source. Following the previous example, this formulation requires \( 100 \times 100 = 10,000 \) dispersion kernel evaluations. This represents a saving of order \( 10^3 \), and it is this relative saving that can be extended to area...
sources appropriate for modelling runways, roads and terminal areas etc, and perhaps to alternative dispersion kernels. (Computational overhead would be expected to reduce this saving by some degree.)

From a regulatory perspective, the area $A_r$ in exceedance of some regulatory concentration limit $\chi_r$ is of interest. For a single point source, we can derive the scaling for this area $A^*_r$ from Eq. (11). Taking $\rho(\theta) = 1/2\pi$ and if $\langle u_r \rangle$ is of the same order as $\langle u^1 \rangle$, then

$$A^*_r = \frac{Q\bar{u}}{\kappa\langle u_r \rangle\chi_r},$$

(12)

This could be derived directly by considering that the concentration given by Calder’s two-dimensional formula is approximately spread over $2\pi R$ on average. It would be expected that $A_r/A^*_r \sim 1$, where $A_r$ is calculated by numerical solution of Eq. (11).

**POINT SOURCE APPLICATION TO HEATHROW**

Figure 1 shows NO$_x$ concentration contours for Heathrow due to aircraft only, as calculated by EDMS as part of PSDH. Figure 2 shows the equivalent plot calculated by direct numerical integration of a Calder-type point source in Eq. (1). Figure 3 shows the Calder-type calculation, but with the approximation relating to lateral dispersion over the long-term made in reaching Eq. (11). The results are capped at 56 µg/m$^3$ for rendering purposes.

Figures 1, 2 and 3 represent ~10 days, ~100 sec and less than 1 sec of run-time respectively on a personal computer. The point source algorithms were implemented in MATLAB 7.4. It can be seen that the point source characterizes the exceedance area at concentrations of regulatory interest. As would be expected, near the point source concentrations are too high, and die away more quickly than area sources. The faster Eq. (11) version of the Calder-type calculation, which makes use of the lateral dispersion averaging approximation, demonstrates little change compared to direct numerical integration. It has a more ragged look due to the lack of lateral Gaussian smoothing and the limited resolution of the wind rose input data. For these calculations a Heathrow wind rose was used, $z_0 = 0.2$ m and $Q = 1768$ tonnes/year (Underwood et al, 2004). Direct use of Eq. (12) gives $A^*_r = 6.2$ km$^2$ for $\chi_r = 50$ µg/m$^3$ and $\langle u_r \rangle = 4.4$ m/s, which would appear to be of the correct order.

**CONCLUDING REMARKS**

The EU regulatory context of airport air quality impacts implies that long-term limit value for NO$_2$ is a primary concern. A framework of simplifying assumptions for urban airports was outlined. A method that directly yields long-term average concentrations with minimal data input requirements was described for point sources. Additional assumptions were made to save up to a factor of $\sim 10^3$ in execution time. This has been extended to area sources, and may
constitute the basis of a rapid operational screening model for airport expansion plans or policy options. Possible extension to account for atmospheric stability involves computing \( \langle u' (\theta) \rangle p(\theta) \), where \( f \) is related to a power-law velocity profile parameter, however neutral conditions remain a good simplifying assumption.

ACKNOWLEDGMENTS
We wish to thank Tom Reynolds, Andreas Schäfer, Marcus Köhler, Lynette Dray and Tony Evans for useful discussions, Cambridge Environmental Research Consultants Ltd for providing wind statistics and Silvana Di Sabatino for providing EMDS results from PSDH.

REFERENCES